Motion from the past – Technical appendix

I. Central streamlines extraction

This appendix describes the algorithm we designed to compute the central streamlines of semicircular canals from a mesh of either osseous or membranous labyrinthine surface. The procedure is decomposed into four main steps:

(i) data organization and completion
(ii) building of an interior cloud of points
(iii) computing the centres of optimal sections
(iv) polygon editing and interpolation

1) Data organization and completion:

Several computations in the following steps involve the selection of spatial neighbours of a given point. In order to reduce computational costs, all 3D data points are reordered in small cubic boxes. Cubic boxes are defined as integer division of the enveloping box, which is the smallest rectangular parallelepiped including all data points. Such a data organization allows finding neighbours of a given point without checking all data points, but only those contained in neighbour boxes.

If not already available, the normal vector and the barycentre of all triangles are also computed at this step. The labyrinthine surface is therefore completed by a set of 6D data $\{X_i, N_i\}$ where X_i is the barycentre and N_i the vector normal to the i^{th} triangle. By convention, the normal vectors are directed towards the inside of the labyrinthine cavity.

2) The interior cloud of points

Ideally, we seek for the cut-locus of the boundary surface of a closed volume, i.e. the locus of points equidistant from 3 points of the surface. Since the surface is not smooth, but approximated by a set of triangles and since the computation of the cut-locus is of the order of n^3 (n is the number of triangles), the exact computation of the cut-locus of boundary surface is not tractable. Instead, the aim of this second step is to provide a reasonable approximation of the cut-locus as the cloud of points which are the centres of maximal inscribed spheres. First, we compute for each triangle the maximal positive scalar ρ_i such that the sphere centred on $C_i = X_i + \rho_i.N_i$ and of radius ρ_i contains no other barycentre than X_j such that the angle between N_j and N_i is greater or equal to 90°. The radius ρ_i is quickly found by dichotomy. The centre Ci and the normal vectors define a plane π_i (which is a crude approximation of an optimal section plane, see below) orthogonal to the cross product $N_i x N_j$.

Once all the triangles have been processed, we refine the centre and radius of each sphere (C_i, ρ_i) with C_i contained in the plane π_i. We look for bigger spheres (C'_i, $\rho'_i > \rho_i$) with C'_i contained in the slice parallel to π_i of thickness δ. If any, we projected their centres back on the plane, and we tested whether the new inscribed sphere (C''_i, ρ''_i) has or not a greater radius than the initial one. We define the interior cloud of points as the set of centre of spheres of maximal radius obtained by repeating this procedure until no radius increase can be further obtained (Fig. 1).

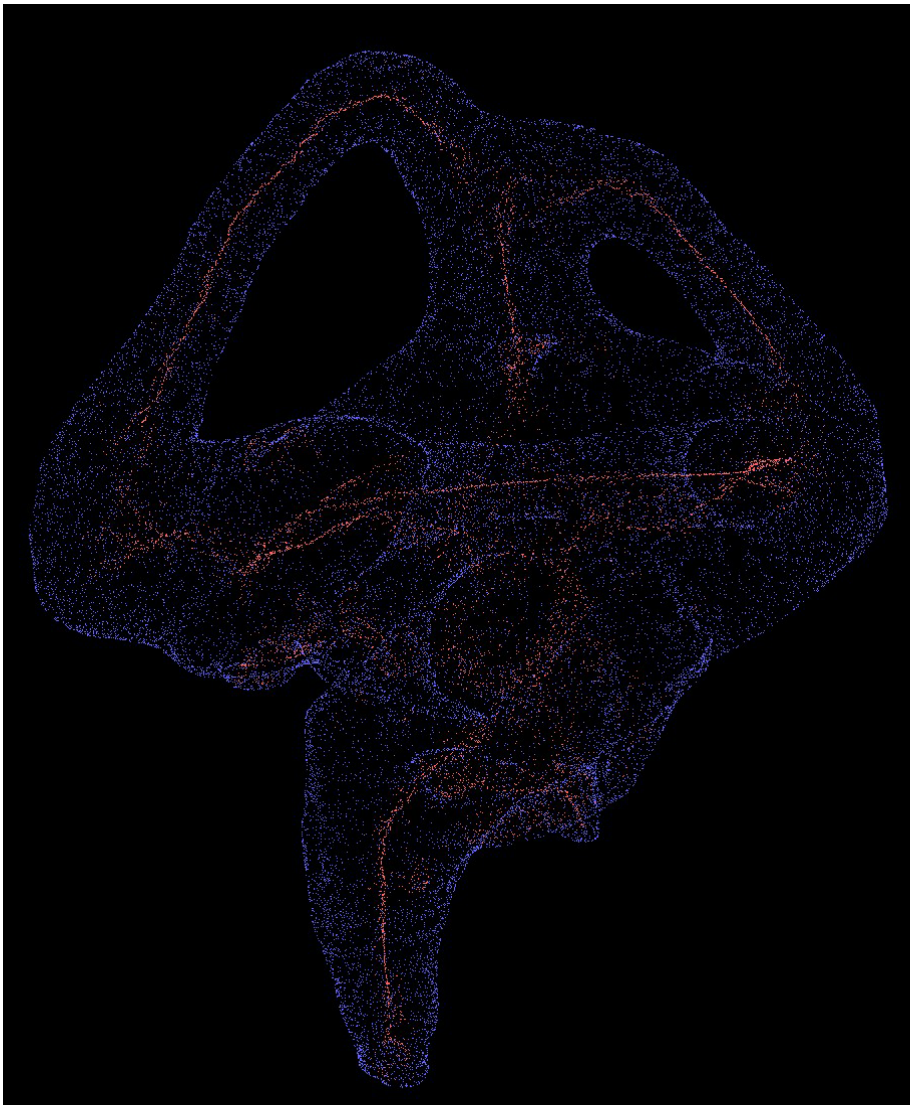

Figure 1: Raw interior cloud of points (in red).

3) The centres of optimal sections

The goal of this step is to provide a more precise evaluation of the canal central streamline, defined as the locus of barycentres of optimal sections. An optimal section is defined as the planar section of minimal area mostly orthogonal to the canal surface. If a canal could be well approximated by a cylinder, the central streamline would merge in its axis. In this case, the maximal inscribed sphere centres obtained in the previous section would accurately specify the central streamline. However, this assumption is far from exact , and, as shown in figure 1, the sphere centres frequently spread away from the apparent axis. The centres of optimal sections are computed with the following procedure. Given an interior point C and a plane π passing through C, one computes the section $S(C, \pi)$ of the mesh as the set of intersections of all the triangles with π which are visible (i.e. not occluded by other triangles) from the point C. The area, the perimeter and the barycentre of $S(C, \pi)$ are also computed. We then define the following iterative process to find the optimal section:

(i) The process is initialized by setting $C(0)$ and $\pi(0)$ as one centre C_i and its associated plane π_i extracted from the interior cloud built in the previous step.

(ii) $C(k+1)$ is computed as the barycentre of the section $S(C(k), \pi(k))$

(iii) The normals of visible intersected triangles are used to build the 3x3 symmetric matrix $M = \Sigma_j N_j^T.N_j$. The eigenvectors and the real positive eigenvalues of M are computed. The normalized eigenvector N with the smallest eigenvalue is the most orthogonal direction to the set of normals, i.e. it minimized $\Sigma_j (N.N_j^T)^2$

(iv) $\pi(k+1)$ is the plane passing through $C(k+1)$ and orthogonal to N

We define the "optimal section" as the one in this sequence which has the minimal area. In practice, a few iterations (about ten) are sufficient to get the optimal section and its barycentre with reasonable accuracy. The figure 2 shows the result of this process.

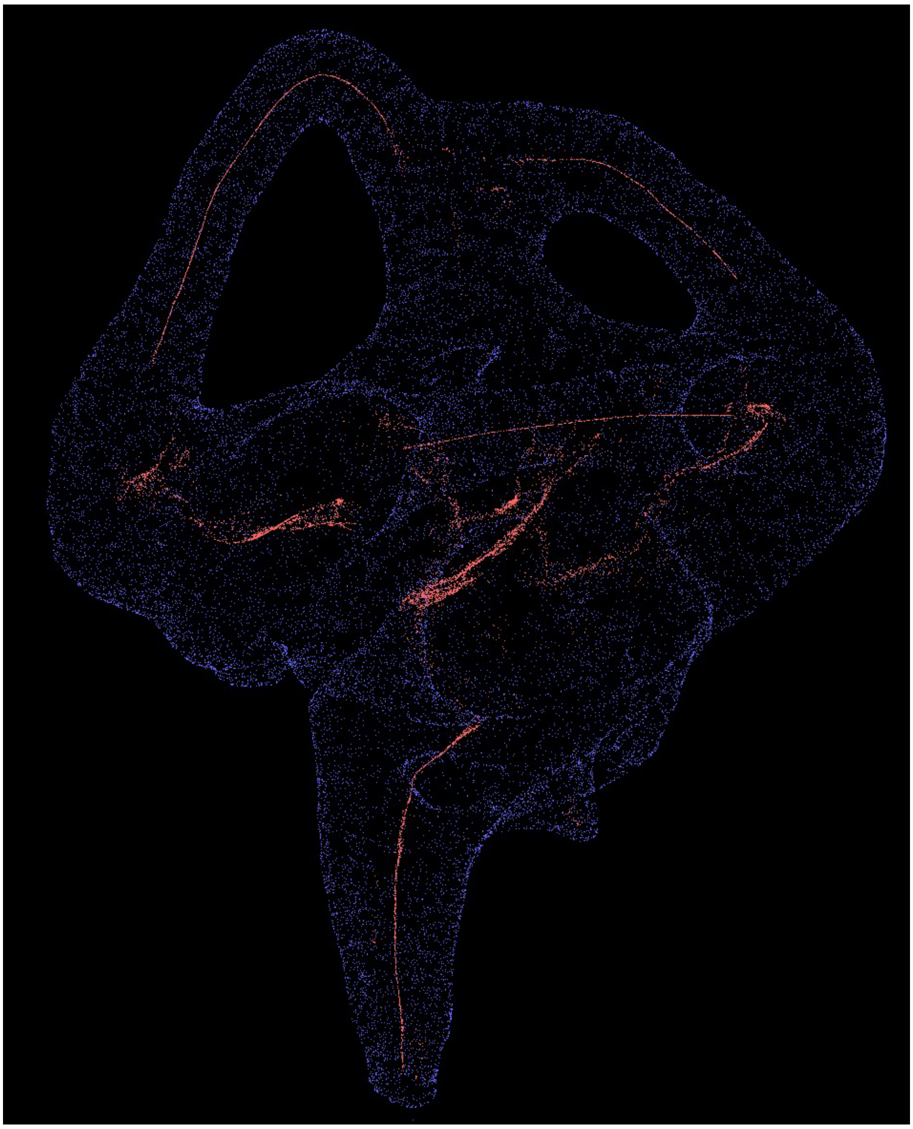

Figure 2: Barycentres of the optimal sections (in red).

4) Polygon edition and interpolation

The objective of this step is to transform a cloud of points, thought to be reasonably close to the main axis of the canals, into a set of sufficiently regular 3D polygons. We start by picking one centre of optimal section obtained in the previous step, which constitutes the first summit P(0) of a new polygon. Then we move along the direction N(0) orthogonal to the optimal section on a small distance δ. The next summit P(1) and the next direction of move N(1) are obtained from the iterative process defined in step 3, using $P(0) + \delta.N(0)$ and N(0) to initialize the centre and the intersection plane. The centres of optimal sections which are crossed during the move are removed. The polygon is built by repeating this process until one of the following stop conditions is reached:

- (i) All the centres of optimal sections have been removed.
- (ii) The area of the optimal section of the last summit exceeds a threshold Amax.
- (iii) The angle between two successive directions of displacement exceeds a threshold Dmax.

Whenever it is possible, the polygon is completed by the opposite one, that is the polygon starting from P(0) and moving in the opposite direction N'(0) = - N(0).

This automatic process provides good results in the thin parts of canals, but has to be manually edited to remove too short polygons or polygons outside the regions of interest. For various reasons due to the morphology of the labyrinths, the automatic procedure does not generate a complete set of polygons for all the semi-circular canals. A complete polygon set is finally generated using 3D cubic B-spline interpolation and straight lines. Two summits from two different polygons are connected by a third order polynomial curve constrained to pass through both summits, to be tangent to both polygons and to have a minimal curvature in the vicinity of both summits. The final result of the whole procedure is in figure 3. On a standard desktop computer, the whole procedure can be performed in less than one hour per labyrinth (manual edition not included).

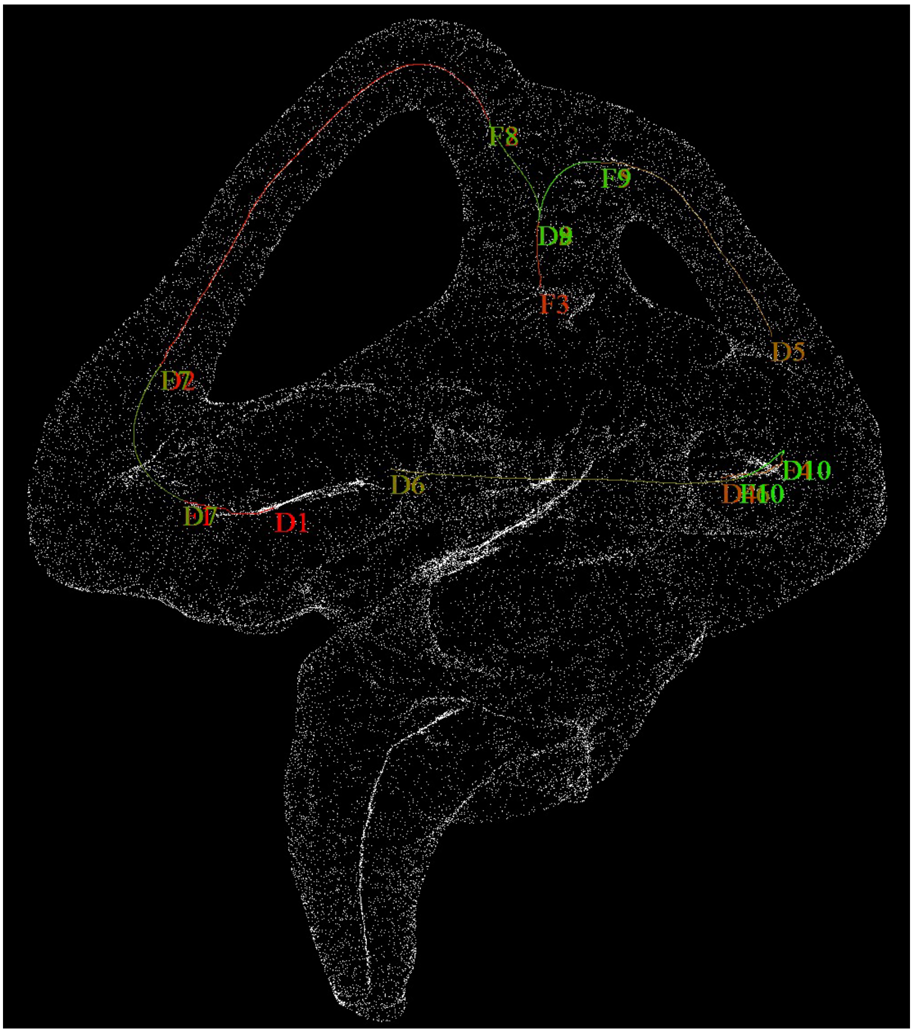

Figure 3: Raw polygons resulting from the entire process.

II. Calculation of referential axes coordinates

1) Calculation of the z axis

In the proposed system, the z axis corresponds to the axis which is perpendicular to the lateral canals synergistic pair plane. The $\vec{z}$ vector is a unit vector and points dorsally, so we have :

$$\vec{z}=\frac{\dfrac{\overrightarrow{X_{Lg}}}{\|\overrightarrow{X_{Lg}}\|}-\dfrac{\overrightarrow{X_{Ld}}}{\|\overrightarrow{X_{Ld}}\|}}{\left\|\dfrac{\overrightarrow{X_{Lg}}}{\|\overrightarrow{X_{Lg}}\|}-\dfrac{\overrightarrow{X_{Ld}}}{\|\overrightarrow{X_{Ld}}\|}\right\|} \qquad (1)$$

2) Calculation of the x axis

The x axis corresponds to the axis which is perpendicular to the coronal plane. The $\vec{x}$ vector is a unit vector and points to the left. Since the plane of bilateral symmetry of the vectors (coronal plane) differs from the plane of bilateral symmetry of the canals (sagittal plane), we have :

$$\vec{x}_1 = \frac{\dfrac{\overrightarrow{X_{Ad}}}{\|\overrightarrow{X_{Ad}}\|} + \dfrac{\overrightarrow{X_{Ag}}}{\|\overrightarrow{X_{Ag}}\|}}{\left\| \dfrac{\overrightarrow{X_{Ad}}}{\|\overrightarrow{X_{Ad}}\|} + \dfrac{\overrightarrow{X_{Ag}}}{\|\overrightarrow{X_{Ag}}\|} \right\|} \times \vec{z} \tag{2}$$

$$\vec{x}_2 = \vec{z} \times \frac{\dfrac{\overrightarrow{X_{Pd}}}{\|\overrightarrow{X_{Pd}}\|} + \dfrac{\overrightarrow{X_{Pg}}}{\|\overrightarrow{X_{Pg}}\|}}{\left\| \dfrac{\overrightarrow{X_{Pd}}}{\|\overrightarrow{X_{Pd}}\|} + \dfrac{\overrightarrow{X_{Pg}}}{\|\overrightarrow{X_{Pg}}\|} \right\|} \tag{3}$$

$$\vec{x} = \frac{\vec{x}_1 + \vec{x}_2}{\|\vec{x}_1 + \vec{x}_2\|} \tag{4}$$

3) Calculation of the y axis

The y axis corresponds to the axis which is perpendicular to the bilateral symmetry plane. The $\vec{y}$ vector is a unit vector and points to the left.

$$\vec{y} = \vec{z} \times \vec{x} \tag{5}$$

After calculation, any object coordinates can be simply converted in the reference system coordinates using passage matrices. Such conversion is presented in figure 4.

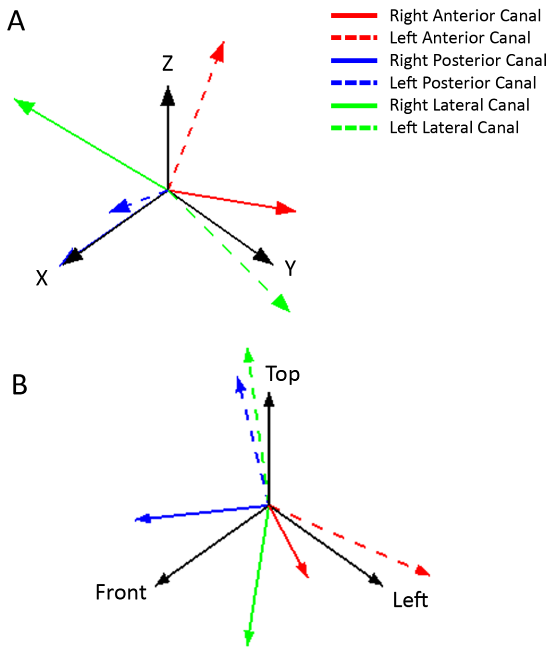

Figure 4: A - SCFS in the scan reference system. **B** – The same vectorial structure in the semicircular canals reference system.

III. Calculation of the response analyses

1) Total response

The total response of the system is calculated in attributing to all spherical coordinates (θ,ϕ), where θ correspond to the inclination and ϕ to the azimuth, the value $RT_{(\theta,\phi)}$ where :

$$RT_{(\theta,\phi)}=\sum \|\overrightarrow{\Omega_{(\theta,\phi)}}.\overrightarrow{X'_n}\| \tag{6}$$

$\overrightarrow{\Omega_{(\theta,\phi)}}$ is an angular acceleration vector with a normalized magnitude and of spherical coordinates (θ,ϕ) and $\overrightarrow{X'_n}$ corresponds to each of the six sensitivity vectors of the system.

2) Global response

The global response of the system is calculated in attributing to all spherical coordinates (θ,ϕ), where θ corresponds to the inclination and ϕ to the azimuth, the value $RG_{(\theta,\phi)}$ where :

$$RG_{(\theta,\phi)}=\sum \overrightarrow{\Omega_{(\theta,\phi)}}.\overrightarrow{X'_n} \tag{7}$$

$\overrightarrow{\Omega_{(\theta,\phi)}}$ is an angular acceleration vector with a normalized magnitude and of spherical coordinates (θ,ϕ) and $\overrightarrow{X'_n}$ corresponds to each of the six sensitivity vectors of the system.